e.g. Let ω be a non-deg 2-form on M s.t. $d\alpha = 0$

pointwise ω_p is a non-deg bi-linear form

"$\therefore$ α is closed"

Then given a function $H: M \longrightarrow \mathbb{R}$, non-deg of ω implies

$$dH \quad \xleftrightarrow[\omega(X_H,-)=-dH]{\omega} \quad X =: X_H$$

(Hamiltonian vector field given by H).

Then the 1-par group of diffeo on M, associated to X_H, is denoted by φ_H^t (called Hamiltonian isotopy).

Then

This can be back to Liouville

① $$L_{X_H}\omega = d\,\iota_{X_H}\omega + \underbrace{\iota_{X_H}d\omega}_{=0} = d\left(\omega(X_H,-)\right) = -d(dH) = 0.$$

so ω does not change at all along X_H, that is, $(\varphi_H^t)^*\omega = \omega$.

② $$L_{X_H}H = dH(X_H) = -\omega(X_H, X_H) = 0$$

so H is constant along its own Hamiltonian vector field.